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GENERALIZED PRODUCT TOPOLOGY

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Abstract. Similarly to Tychonoff product, we introduce the concept of generalized product topology which is different from the notion of product of generalized topologies in [Á. Császár, *Acta Math. Hungar.* 123 (2009), 127–132] for generalized topology and obtain some properties about it. Besides, we prove that connectedness, σ-connectedness and α-connectedness are all preserved under this product.

1. Introduction and preliminaries

In the past years, several weak forms of open sets have been studied. Recently, Á. Császár founded the theory of generalized topology in [1-8], studying the extremely elementary character of these classes. It is well known that *Tychonoff product* plays an important role in topological spaces. Motivated by these, we shall investigate into 'generalized product topology' on generalized topological spaces.

Let X be a set, and denote $\exp X$ the power set of X. We call a class $\lambda \subset \exp X$ a *generalized topology* (briefly GT) [2] on X if $\emptyset \in \lambda$ and any union of elements of λ belongs to λ. A set with a GT is said to be a *generalized topological space* (briefly GTS) [2]. For a GTS (X, λ), the elements of λ are called λ-*open sets* and the complements of λ-open sets are called λ-*closed sets*. For any $x \in X$, put $\mathscr{N}(x) = \{A \in \lambda : x \in A\}$. For $A \subset X$, we denote by cA the intersection of all λ-closed sets containing A and by iA the union of all λ-open sets contained in A. A set $A \subset X$ is said to be λ-*semi-open* (resp. λ-*preopen*, λ-α-*open*, λ-β-*open*) [4] if $A \subset ciA$ (resp. $A \subset icA$, $A \subset iciA$, $A \subset cicA$). We denote by $\sigma(\lambda)$ (resp. $\pi(\lambda)$, $\alpha(\lambda)$, $\beta(\lambda)$) the class of all λ-semi-open sets (resp. λ-preopen sets, λ-α-open sets, λ-β-open sets). Obviously $\lambda \subset \alpha(\lambda) \subset \sigma(\lambda) \subset \beta(\lambda)$ and $\alpha(\lambda) \subset \pi(\lambda) \subset \beta(\lambda)$.

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A family $\lambda_b \subset \lambda$ is called a *base* for a GTS (X, λ) if every non-empty λ-open subset of X can be represented as the union of a subfamily of λ_b. We denote by $\mathscr{B}(\lambda)$ of all bases of GTS (X, λ).

According to the definition of cA, similarly to the proof of [7, Proposition 1.1.1], it is not difficult to prove the following conclusion:

Lemma 1.1. *For any $A \subset X$, the following conditions are equivalent*:

1-1) $x \in cA$;

1-2) *For any $B \in \mathscr{N}(x)$, we have $B \cap A \neq \emptyset$;*

1-3) *There exists some $\lambda_b \in \mathscr{B}(\lambda)$ such that for any $B \in \mathscr{N}(x) \cap \lambda_b$, $B \cap A \neq \emptyset$.*

Let (X, λ) and (Y, λ') be two generalized topological spaces; a map $f : X \longrightarrow Y$ is called *continuous* (called (λ, λ')-*continuous* in [9]) if $f^{-1}(A) \in \lambda$ for any $A \in \lambda'$.

A GTS (X, λ) is said to be *connected* (called γ-*connected* in [3]) if there are no nonempty disjoint sets $U, V \in \lambda$ such that $U \cup V = X$.

A GTS (X, λ) is called α-*connected* (resp. σ-*connected*, π-*connected*, β-*connected*) [11] if $(X, \alpha(\lambda))$ (resp. $(X, \sigma(\lambda))$, $(X, \pi(\lambda))$, $(X, \beta(\lambda))$) is connected.

It is easy to see from the definition that

$$\beta\text{-connected} \Rightarrow \sigma\text{-connected} \Rightarrow \alpha\text{-connected} \Rightarrow \text{connected}$$

and

$$\beta\text{-connected} \Rightarrow \pi\text{-connected} \Rightarrow \alpha\text{-connected.}$$

In [11], the following result was proved:

Lemma 1.2 ([11])**.** *For a GTS (X, λ), (X, λ) is α-connected if and only if (X, λ) is connected.*

Suppose we are given a set X, a family $\{(Y_s, \lambda_s)\}_{s \in \Gamma}$ of GTS and a family of maps $\{f_s\}_{s \in \Gamma}$, where f_s is a map of X to Y_s. It is easy to see that the GT

$$\lambda = \left\{ \cup A : A \subset \{ f_s^{-1}(A_s) : A_s \in \lambda_s, s \in \Gamma \} \right\} \tag{1}$$

is the coarsest GT that makes all the f_s's continuous. This GT is called the *GT generated by the family* $\{f_s\}_{s \in \Gamma}$ of maps.

2. Generalized connectedness under product

Similarly to Tychonoff product which can be found in [10, Section 2.3], now we introduce *generalized product* for GTS. Suppose we are given a family of GTS $\{(X_s, \lambda_s)\}_{s \in \Gamma}$; consider the Cartesian product $X = \prod_{s \in \Gamma} X_s$ and the family of maps p_s, where p_s assigns to the point $x = \{x_s\} \in \prod_{s \in \Gamma} X_s$ its sth coordinate $x_s \in X_s$. The set $X = \prod_{s \in \Gamma} X_s$ with the GT $\prod_{s \in \Gamma} \lambda_s$ generated by the family of $\{p_s\}_{s \in \Gamma}$ is called the *generalized product topology space* (briefly GPTS) and $\prod_{s \in \Gamma} \lambda_s$ is called the *generalized product topology* on $\prod_{s \in \Gamma} X_s$ (briefly GPT); The map $p_s : \prod_{s \in \Gamma} X_s \longrightarrow X_s$ is called the *projection of* $\prod_{s \in \Gamma} X_s$ *onto* X_s. Clearly, the GPTS is usually different from the product of generalized topologies in [8] for generalized topology. Put the set $\mathscr{B}^*\left(\prod_{s\in\Gamma}\lambda_s\right)=\left\{p_s^{-1}(B_s):B_s\in\lambda_s,s\in\Gamma\right\}$.

Proposition 2.1. *For a GPTS* $\left(\prod_{s\in\Gamma}X_s,\prod_{s\in\Gamma}\lambda_s\right)$,

$$\mathscr{B}^*\left(\prod_{s\in\Gamma}\lambda_s\right)\in\mathscr{B}(\prod_{s\in\Gamma}\lambda_s).$$

Proof. It is clear that $\mathscr{B}^*\left(\prod_{s\in\Gamma}\lambda_s\right)=\left\{p_s^{-1}(B_s):B_s\in\lambda_s,s\in\Gamma\right\}\subset\prod_{s\in\Gamma}\lambda_s$. Combining this with (1), the proof is completed. □

Proposition 2.2. $c\left(\prod_{s\in\Gamma}A_s\right)=\prod_{s\in\Gamma}cA_s$;

$$i\left(\prod_{s\in\Gamma}A_s\right)=\begin{cases}\emptyset, & |\{s\in\Gamma:A_s\neq X_s\}|\geq 2,\\ \prod\limits_{s\in\{s\in\Gamma:A_s\neq X_s\}}iA_s\times\prod\limits_{s\in\Gamma-\{s\in\Gamma:A_s\neq X_s\}}X_s, & |\{s\in\Gamma:A_s\neq X_s\}|=1,\\ \prod_{s\in\Gamma}X_s, & |\{s\in\Gamma:A_s\neq X_s\}|=0.\end{cases}$$

Proof. Applying Lemma 1.1, we have

$$\begin{aligned}&c\left(\prod_{s\in\Gamma}A_s\right)\\&=\left\{x=\{x_s\}\in\prod_{s\in\Gamma}X_s:\forall B\in\mathscr{N}(x)\cap\mathscr{B}^*\left(\prod_{s\in\Gamma}\lambda_s\right),B\cap\left(\prod_{s\in\Gamma}A_s\right)\neq\emptyset\right\}\\&=\left\{x=\{x_s\}\in\prod_{s\in\Gamma}X_s:\forall s\in\Gamma,\forall B_s\in\mathscr{N}(x_s),B_s\cap A_s\neq\emptyset\right\}\\&=\prod_{s\in\Gamma}cA_s.\end{aligned}$$

The second equation is easy to verify. □

It can be verified that

$$\begin{aligned}\alpha\left(\prod_{s\in\Gamma}\lambda_s\right)\supset\prod_{s\in\Gamma}\alpha(\lambda_s),\quad\sigma\left(\prod_{s\in\Gamma}\lambda_s\right)\supset\prod_{s\in\Gamma}\sigma(\lambda_s),\\ \pi\left(\prod_{s\in\Gamma}\lambda_s\right)\supset\prod_{s\in\Gamma}\pi(\lambda_s),\quad\beta\left(\prod_{s\in\Gamma}\lambda_s\right)\supset\prod_{s\in\Gamma}\beta(\lambda_s).\end{aligned}\tag{2}$$

At the end of this paper, we shall use an example to show that the inclusion of (2) can hold strictly.

Proposition 2.3. *For a GPTS* $\left(\prod_{s\in\Gamma} X_s, \prod_{s\in\Gamma} \lambda_s\right)$,

$$c(\cup_{s\in\Gamma} p_s^{-1}(A_s)) = \prod_{s\in\Gamma} X_s,$$

where $\emptyset \neq A_s \subset X_s$ *for* $s \in \Gamma$.

Proof. Choose arbitrarily $x = \{x_s\} \in \prod_{s\in\Gamma} X_s$. For any $B \in \mathscr{N}(x)$, we have that there exist $s_0 \in \Gamma$ and $\emptyset \neq B_{s_0} \in \lambda_{s_0}$ such that $x \in p_{s_0}^{-1}(B_{s_0}) \subset B$. So $B \cap (\cup_{s\in\Gamma} p_s^{-1}(A_s)) = \cup_{s\in\Gamma}(B \cap p_s^{-1}(A_s)) \supset \cup_{s\in\Gamma}(p_{s_0}^{-1}(B_{s_0}) \cap p_s^{-1}(A_s)) \supset \cup_{s\in\Gamma-\{s_0\}}(p_{s_0}^{-1}(B_{s_0}) \cap p_s^{-1}(A_s)) \neq \emptyset$ as each $A_s \neq \emptyset$. Combining this with Lemma 1.1, it follows that $x \in c(\cup_{s\in\Gamma} p_s^{-1}(A_s))$.

Hence $c(\cup_{s\in\Gamma} p_s^{-1}(A_s)) = \prod_{s\in\Gamma} X_s$. □

Theorem 2.4. *The GPTS* $\left(\prod_{s\in\Gamma} X_s, \prod_{s\in\Gamma} \lambda_s\right)$ *is connected if and only if all spaces* (X_s, λ_s) *are connected.*

Proof. Necessity. Suppose that there exists some $s_0 \in \Gamma$ such that (X_{s_0}, λ_{s_0}) is not connected. Then there exist nonempty disjoint subsets $A_{s_0}, B_{s_0} \in \lambda_{s_0}$ such that $A_{s_0} \cup B_{s_0} = X_{s_0}$. This implies that nonempty $\prod_{s\in\Gamma} \lambda_s$-open sets $p_{s_0}^{-1}(A_{s_0})$ and $p_{s_0}^{-1}(B_{s_0})$ satisfy $p_{s_0}^{-1}(A_{s_0}) \cap p_{s_0}^{-1}(B_{s_0}) = \emptyset$ and $p_{s_0}^{-1}(A_{s_0}) \cup p_{s_0}^{-1}(B_{s_0}) = \prod_{s\in\Gamma} X_s$. So $\left(\prod_{s\in\Gamma} X_s, \prod_{s\in\Gamma} \lambda_s\right)$ is not connected.

Sufficiency. Suppose that $\left(\prod_{s\in\Gamma} X_s, \prod_{s\in\Gamma} \lambda_s\right)$ is not connected. Then there exist nonempty disjoint subsets $A, B \in \prod_{s\in\Gamma} \lambda_s$ such that $A \cup B = \prod_{s\in\Gamma} X_s$. Without loss of generality, we may assume that $A = \cup_{s\in\Gamma'\subset\Gamma} p_s^{-1}(A_s)$, where $\emptyset \neq A_s \in \lambda_s$ for $s \in \Gamma'$.

Now we assert that $|\Gamma'| = 1$.

Obviously, $\Gamma' \neq \emptyset$ as $A \neq \emptyset$. If $|\Gamma'| > 1$, we have that there exist $s_1 \neq s_2 \in \Gamma'$ such that $p_{s_1}^{-1}(A_{s_1}) \cup p_{s_2}^{-1}(A_{s_2}) \subset A$. As $A \neq \prod_{s\in\Gamma} X_s$ and $A \cup B = \prod_{s\in\Gamma} X_s$, then $A_{s_1} \neq X_{s_1}$, $A_{s_2} \neq X_{s_2}$ and $B = \prod_{s\in\Gamma} X_s - A \subset \prod_{s\in\Gamma} X_s - (p_{s_1}^{-1}(A_{s_1}) \cup p_{s_2}^{-1}(A_{s_2})) = p_{s_1}^{-1}(X_{s_1} - A_{s_1}) \cap p_{s_2}^{-1}(X_{s_2} - A_{s_2})$. Applying Proposition 2.2, it follows that $B = iB \subset i(p_{s_1}^{-1}(X_{s_1} - A_{s_1}) \cap p_{s_2}^{-1}(X_{s_2} - A_{s_2})) = i(A_{s_1} \times A_{s_2} \times \prod_{s\in\Gamma-\{s_1,s_2\}} X_s) = \emptyset$, which is a contradiction. Therefore $|\Gamma'| = 1$.

The set $\Gamma' = \{s_1\}$, then there exists $\emptyset \neq A_{s_1} \in \lambda_{s_1}$ such that $A = p_{s_1}^{-1}(A_{s_1})$. So $B = \prod_{s\in\Gamma} X_s - A = p_{s_1}^{-1}(X_{s_1} - A_{s_1}) \in \prod_{s\in\Gamma} \lambda_s$. This leads with the construction of $\prod_{s\in\Gamma} \lambda_s$ to that $\emptyset \neq X_{s_1} - A_{s_1} \in \lambda_{s_1}$. Hence (X_{s_1}, λ_{s_1}) is not connected. □

Theorem 2.5. *The GPTS* $\left(\prod_{s\in\Gamma} X_s, \prod_{s\in\Gamma} \lambda_s\right)$ *is* σ*-connected if and only if all spaces* (X_s, λ_s) *are* σ*-connected.*

Proof. Necessity. Suppose that there exists some $s_0 \in \Gamma$ such that (X_{s_0}, λ_{s_0}) is not σ-connected. Then there exist nonempty disjoint subsets $A_{s_0}, B_{s_0} \in \sigma(\lambda_{s_0})$ such that $A_{s_0} \cup B_{s_0} = X_{s_0}$. This implies that nonempty sets $p_{s_0}^{-1}(A_{s_0})$ and $p_{s_0}^{-1}(B_{s_0})$ satisfy $p_{s_0}^{-1}(A_{s_0}) \cap p_{s_0}^{-1}(B_{s_0}) = \emptyset$ and $p_{s_0}^{-1}(A_{s_0}) \cup p_{s_0}^{-1}(B_{s_0}) = \prod_{s\in\Gamma} X_s$.

Applying Proposition 2.2, noting the fact that $A_{s_0}, B_{s_0} \in \sigma(\lambda_{s_0})$, we have

$$ci(p_{s_0}^{-1}(A_{s_0})) = c(p_{s_0}^{-1}(iA_{s_0})) = p_{s_0}^{-1}(ciA_{s_0}) \supset p_{s_0}^{-1}(A_{s_0}),$$

and

$$ci(p_{s_0}^{-1}(B_{s_0})) = c(p_{s_0}^{-1}(iB_{s_0})) = p_{s_0}^{-1}(ciB_{s_0}) \supset p_{s_0}^{-1}(B_{s_0}).$$

So $\left(\prod_{s\in\Gamma} X_s, \prod_{s\in\Gamma} \lambda_s\right)$ is not σ-connected.

Sufficiency. Suppose that $\left(\prod_{s\in\Gamma} X_s, \prod_{s\in\Gamma} \lambda_s\right)$ is not σ-connected. Then there exist nonempty disjoint subsets $A, B \in \sigma\left(\prod_{s\in\Gamma} \lambda_s\right)$ such that $A \cup B = \prod_{s\in\Gamma} X_s$. As each (X_s, λ_s) is σ-connected, we know from [8, Theorem 2.3] that $X_s = \cup\lambda_s \in \lambda_s$. This implies that $\prod_{s\in\Gamma} X_s \in \prod_{s\in\Gamma} \lambda_s$, i.e., $c\emptyset = \emptyset$. So we have $ciA \supset A \supset iA \neq \emptyset$ and $ciB \supset B \supset iB \neq \emptyset$. Thus there exist $s_1 \in \Gamma$ and $\emptyset \neq A_{s_1} \in \lambda_{s_1}$ such that $A \supset iA \supset p_{s_1}^{-1}(A_{s_1})$. Therefore $B = \prod_{s\in\Gamma} X_s - A \subset \prod_{s\in\Gamma} X_s - p_{s_1}^{-1}(A_{s_1}) = p_{s_1}^{-1}(X_{s_1} - A_{s_1})$. Similarly, we have that there exists $\emptyset \neq B_{s_1} \in \lambda_{s_1}$ such that

$$p_{s_1}^{-1}(A_{s_1}) \subset iA \subset A \subset ciA \subset p_{s_1}^{-1}(X_{s_1} - B_{s_1}),$$

and

$$p_{s_1}^{-1}(B_{s_1}) \subset iB \subset B \subset ciB \subset p_{s_1}^{-1}(X_{s_1} - A_{s_1}).$$

Now we assert that for any $x = \{x_s\} \in iA$, there exists some $A(x) \in \mathscr{N}(x_{s_1})$ such that $x \in p_{s_1}^{-1}(A(x)) \subset A$.

Indeed, if there exists some $x = \{x_s\} \in iA$ such that $p_{s_1}^{-1}(D) \nsubseteq A$ holds for any $D \in \mathscr{N}(x_{s_1})$. Noting that fact that $\left\{p_s^{-1}(B_s) : B_s \in \lambda_s, s \in \Gamma\right\} \in \mathscr{B}\left(\prod_{s\in\Gamma} \lambda_s\right)$, we have that there exists $s_1 \neq s_2 \in \Gamma$ and $A_{s_2} \in \lambda_{s_2}$ such that $x \in p_{s_2}^{-1}(A_{s_2}) \subset A \subset p_{s_1}^{-1}(X_{s_1} - B_{s_1})$. So $p_{s_2}^{-1}(A_{s_2}) \subset p_{s_1}^{-1}(X_{s_1} - B_{s_1})$, which is a contradiction as $s_1 \neq s_2$.

Thus $iA = \cup_{x\in iA} p_{s_1}^{-1}(A(x)) = p_{s_1}^{-1}(\cup_{x\in iA} A(x))$.

Similarly, we know that there exist nonempty subsets $\mathcal{A}_{s_1}, \mathcal{B}_{s_1} \in \lambda_{s_1}$ such that $iA = p_{s_1}^{-1}(\mathcal{A}_{s_1})$ and $iB = p_{s_1}^{-1}(\mathcal{B}_{s_1})$. As $\emptyset = A \cap B \supset iA \cap iB = p_{s_1}^{-1}(\mathcal{A}_{s_1} \cap \mathcal{B}_{s_1})$, we have $\mathcal{A}_{s_1} \subset X_{s_1} - \mathcal{B}_{s_1}$, then $X_{s_1} - c\mathcal{A}_{s_1} \supset \mathcal{B}_{s_1} \neq \emptyset$. It follows from $A \cup B = \prod_{s\in\Gamma} X_s$ and Proposition 2.2 that $ciB = cp_{s_1}^{-1}(\mathcal{B}_{s_1}) = p_{s_1}^{-1}(c\mathcal{B}_{s_1}) \supset B \supset \prod_{s\in\Gamma} X_s - A \supset \prod_{s\in\Gamma} X_s - ciA = p_{s_1}^{-1}(X_{s_1} - c\mathcal{A}_{s_1})$, so $c\mathcal{B}_{s_1} \supset X_{s_1} - c\mathcal{A}_{s_1}$.

The set $C = c\mathcal{A}_{s_1}$ and $D = X_{s_1} - c\mathcal{A}_{s_1}$. Clearly $C \cup D = X_{s_1}$ and $C \cap D = \emptyset$. Meanwhile, we have $ciC \supset ci\mathcal{A}_{s_1} = c\mathcal{A}_{s_1} = C$ and $ciD \supset ci\mathcal{B}_{s_1} = c\mathcal{B}_{s_1} \supset D$. Hence (X_{s_1}, λ_{s_1}) is not σ-connected as both C and D are nonempty. □

Theorem 2.6. *Given a family of GTS $\{(X_s, \lambda_s)\}_{s\in\Gamma}$, the following are equivalent*:

6-1) $\left(\prod_{s\in\Gamma} X_s, \prod_{s\in\Gamma} \lambda_s\right)$ *is α-connected*;
6-2) $\left(\prod_{s\in\Gamma} X_s, \prod_{s\in\Gamma} \lambda_s\right)$ *is connected*;
6-3) *All spaces (X_s, λ_s) are α-connected*;
6-4) *All spaces (X_s, λ_s) are connected.*

Proof. Applying Lemma 1.2 and Theorem 2.4, it holds trivially. □

Applying Proposition 2.2, it follows that for any $s_0 \in \Gamma$ and any $A_{s_0} \subset X_{s_0}$,

$$ic(p_{s_0}^{-1}(A_{s_0})) = i(p_{s_0}^{-1}(cA_{s_0})) = p_{s_0}^{-1}(icA_{s_0}),$$

and

$$cic(p_{s_0}^{-1}(A_{s_0})) = ci(p_{s_0}^{-1}(cA_{s_0})) = c(p_{s_0}^{-1}(icA_{s_0})) = p_{s_0}^{-1}(cicA_{s_0}).$$

Similarly to the proof of Theorem 2.5, the following theorem holds trivially:

Theorem 2.7. *All spaces (X_s, λ_s) are π-connected (resp. β-connected) provided that the GPTS $\left(\prod_{s\in\Gamma} X_s, \prod_{s\in\Gamma} \lambda_s\right)$ is π-connected (resp. β-connected).*

Being the end of this paper, we shall use an example which is similar to the construction of Example 2.5 in [12] to show that

(1) The inverse of Theorem 2.7 is not correct;

(2) The inclusion of (2) can hold strictly.

Example 2.8. Let $X_1 = X_2 = \{a, b\}$ and $\lambda_1 = \lambda_2 = \{\emptyset, \{a\}, \{a, b\}\}$. Clearly the GTP (X_1, λ_1) and (X_2, λ_2) are connected and

$$\lambda_1 \times \lambda_2 = \{\emptyset, \{(a, a), (a, b)\}, \{(a, a), (b, a)\}, \{(a, a), (a, b), (b, a)\}, X_1 \times X_2\}.$$

As $cic\{b\} = ci\{b\} = c\emptyset = \emptyset$, we have $\beta(\lambda_i) = \lambda_i$. So (X_1, λ_1) and (X_2, λ_2) are β-connected (thus π-connected) and

$$\sigma(\lambda_1) \times \sigma(\lambda_2) = \pi(\lambda_1) \times \pi(\lambda_2) = \alpha(\lambda_1) \times \alpha(\lambda_2) = \beta(\lambda_1) \times \beta(\lambda_2) = \lambda_1 \times \lambda_2.$$

Take $A = \{(a, a)\}$ and $B = X_1 \times X_2 - A = \{(a, b), (b, a), (b, b)\}$. Applying Lemma 1.1 and Proposition 2.3, it is easy to see that $icA = icB = X_1 \times X_2$, i.e., $A, B \in \pi(\lambda_1 \times \lambda_2)$. So the GPTS $(X_1 \times X_2, \lambda_1 \times \lambda_2)$ is not π-connected (thus not β-connected).

Choose $D = \{(a, a), (a, b), (b, a)\}$. We know from Proposition 2.3 that $iciD = X_1 \times X_2 \supset D$. This implies that $D \in \alpha(\lambda_1 \times \lambda_2) - \alpha(\lambda_1) \times \alpha(\lambda_2)$. Thus $D \in \sigma(\lambda_1 \times \lambda_2) - \sigma(\lambda_1) \times \sigma(\lambda_2)$, $D \in \pi(\lambda_1 \times \lambda_2) - \pi(\lambda_1) \times \pi(\lambda_2)$, $D \in \beta(\lambda_1 \times \lambda_2) - \beta(\lambda_1) \times \beta(\lambda_2)$.

Hence $\alpha(\lambda_1 \times \lambda_2) \supsetneqq \alpha(\lambda_1) \times \alpha(\lambda_2)$, $\sigma(\lambda_1 \times \lambda_2) \supsetneqq \sigma(\lambda_1) \times \sigma(\lambda_2)$, $\pi(\lambda_1 \times \lambda_2) \supsetneqq \pi(\lambda_1) \times \pi(\lambda_2)$, $\beta(\lambda_1 \times \lambda_2) \supsetneqq \beta(\lambda_1) \times \beta(\lambda_2)$.

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